Data were also acquired at a −4-deg angle of attack, which allowed more complete measurement of pressure side heat transfer values. Comparing the −4-deg (pressure side) data with the +4-deg (suction side) data showed that for the roughened airfoil the pressure side experienced lower heat transfer than the suction side.

In computer codes, heat transfer from airfoils is often estimated by using cylinder-in-crossflow heat transfer values in the leading edge region and flat plate heat transfer values farther aft. Figure 6 shows the IRT data for the dense 2, 0-deg, without spray condition compared with the cylinder and flat plate heat transfer values. The heat transfer in the stagnation region for the dense 2 roughened airfoil agrees fairly well with Frossling's[5] smooth cylinder laminar flow solution. Moving downstream on the airfoil, the heat transfer drastically increases, reaching a maximum level near s/c of 0.035, and then decreases to a level fairly consistent with turbulent flow flat plate heat transfer values.[6] The measured Frossling numbers at specific Reynolds numbers are somewhat higher than their respective flat plate turbulent values. However, the higher measured heat transfer may be due to the increase in surface area caused by the roughness elements (3–7% increase on each gauge for the dense roughness patterns) that was not taken into account in the data analysis. It may be mentioned here that the maximum heat transfer is in the same general region, if slightly aft, of ice horn growth observed during glaze ice accretion.[7]

Conclusions

Local heat transfer measurements from a roughened NACA 0012 airfoil were successfully obtained in flight and in the NASA Lewis icing research tunnel using the method and apparatus described in this work. Major conclusions resulting from this study are as follows.

1) The addition of roughness to the airfoil surface drastically increased the heat transfer downstream of stagnation. The roughness elements disturbed the laminar boundary-layer flow and in some cases caused a transition to turbulent flow.

2) Comparison of the flight and tunnel roughened surface data showed that the general effect of increased turbulence was a slight increase in heat transfer, especially at the higher Reynolds numbers.

3) Generally, the roughened surface airfoil cases showed the suction side heat transfer monotonically increasing with angle of attack.

Acknowledgment

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Incompressible Steady Aerodynamics Using a Standard Finite Element Code

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Nomenclature

A = nodal area
n = normal vector
S_B = surface of the body
S_C = surface of the branch cut
S_E = any external surface including the body
V_∞ = asymptotic freestream velocity
∇^2 = Laplacian operator
ϕ = velocity potential

Subscripts

$(\,)_u$ = values of ( ) at the upper side of the trailing edge
$(\,)_l$ = values of ( ) at the lower side of the trailing edge

I. Introduction

IN the last decade, great attention has been devoted to the solution of the wave equation using the finite element technique.[1,2] The use of a finite element structural code for general scalar fields can be approached on the basis of an analogy between structural equations of elasticity and those for the relevant scalar fields. For lifting aerodynamic fields, the same approach may be used, but a new problem due to the circulation must be solved. In this Note, a simple way to solve steady incompressible aerodynamic fields around a body is presented. The work extends the common approach for the scalar fields using the standard structural finite element code to lifting flowfields. One of the greatest advantages of this approach is the possibility of using the standard preprocessing tools developed for structural problems to aerodynamic problems. The described approach can be used for general three-dimensional bodies, or multicomponents airfoils, due to the feasibility of realizing an unstructured mesh, as those required by the finite element technique.

II. Theoretical Formulation

The classical aerodynamic problem is to develop the steady, nonviscous, incompressible aerodynamic field around a body. The governing field equation is

$$\nabla^2\phi = 0 \tag{1}$$

with the boundary conditions (Fig.1) as follows.

The body is a streamline:

$$\frac{\partial\phi}{\partial n} = 0 \quad \text{over } S_B \tag{2}$$

Asymptotic condition:

$$\frac{\partial\phi}{\partial n} = V_\infty \cdot n \quad \text{over } S_E \tag{3}$$

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Furthermore, the Kutta's condition must be imposed at the trailing edge of the airfoil to ensure the uniqueness of the lifting solution.[3]

In Ref. 4, a technique using linear superposition of two fields is suggested to calculate the correct value of the circulation of the velocity circulation Γ. It assumes that the unknown potential ϕ is represented by the following sum:

$$\phi = \Gamma\phi_0 + \phi_1 \tag{4}$$

where ϕ_0 is the solution of a unit circulation field. This was obtained by solving Eq. (1), introducing in the domain a branch cut to get a simple connected domain. The boundary condition is

$$\Delta\phi_0 = 1 \quad \text{over } S_C \tag{5}$$

The ϕ_1 field is the solution of Eq. (1), with the boundary condition along S_E from Eq. (3), and imposing the continuity across the branch cut. Both of the fields must satisfy condition (2).

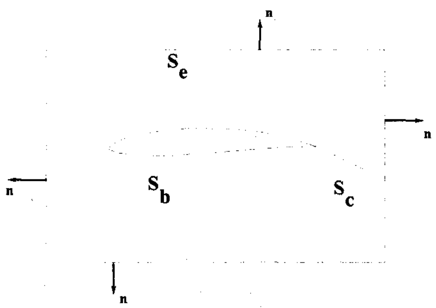

Fig. 1 Boundary of the domain.

Imposing the Kutta's condition at the trailing edge,

$$V_x|_u = V_x|_l$$

$$\left(\frac{\partial\phi}{\partial x}\right)_u = \left(\frac{\partial\phi}{\partial x}\right)_l$$

$$\Gamma\left(\frac{\partial\phi_0}{\partial x}\right)_u + \left(\frac{\partial\phi_1}{\partial x}\right)_u = \Gamma\left(\frac{\partial\phi_0}{\partial x}\right)_l + \left(\frac{\partial\phi_1}{\partial x}\right)_l$$

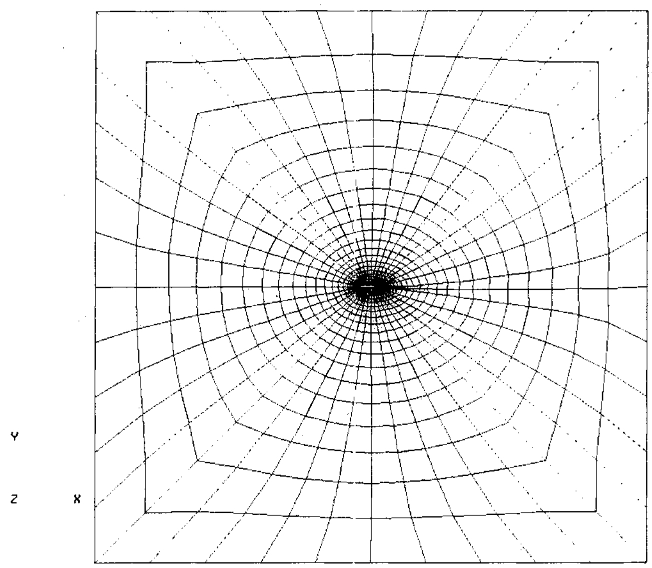

Fig. 2 Finite element mesh.

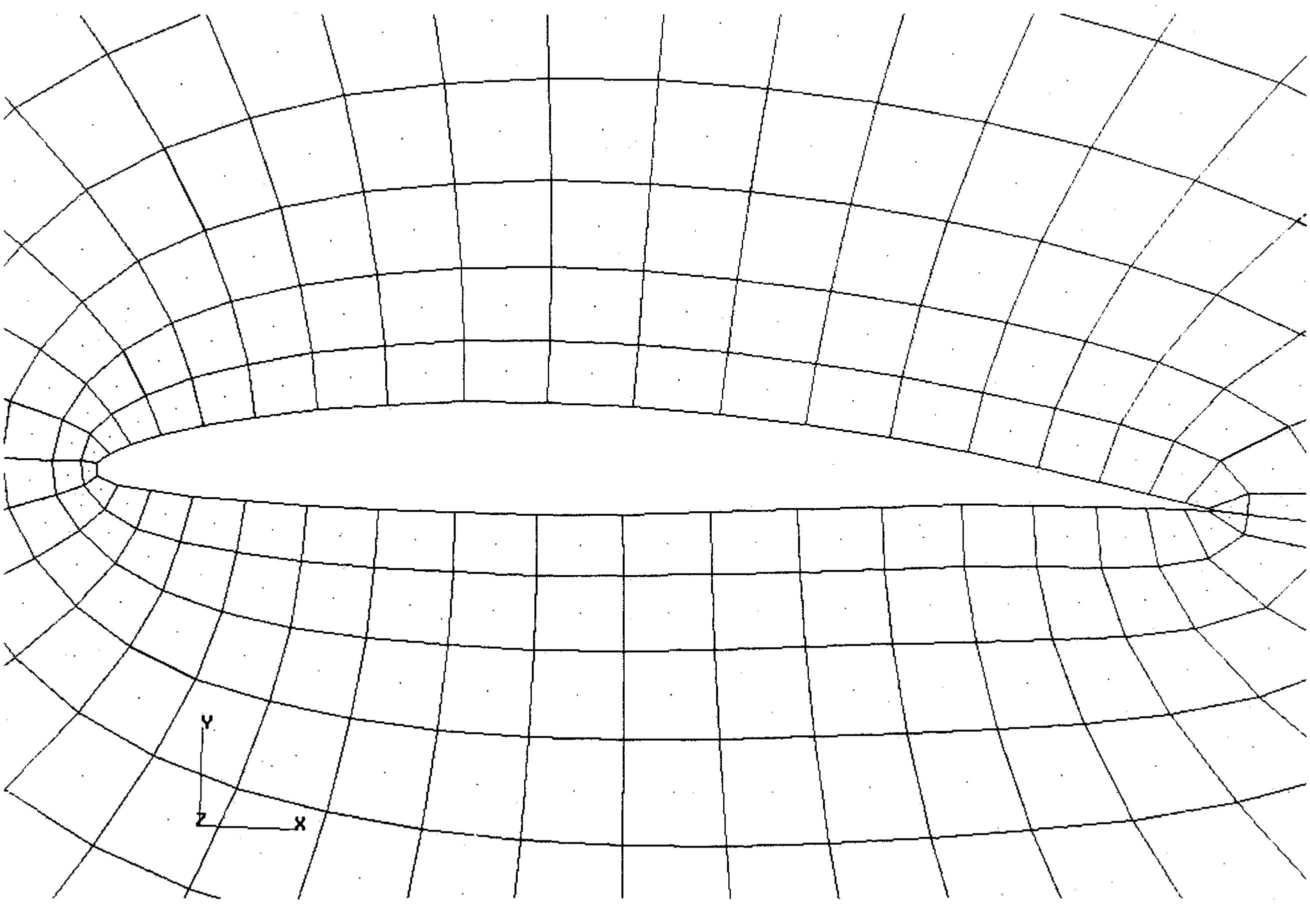

Fig. 3 Zoom of the mesh around the airfoil NACA 64_A410.

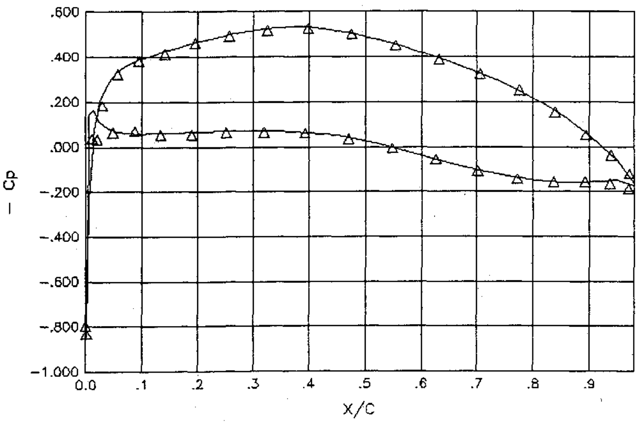

Fig. 4 Pressure coefficient distribution ($\alpha = 0$ deg): — panel method; △ finite element method.

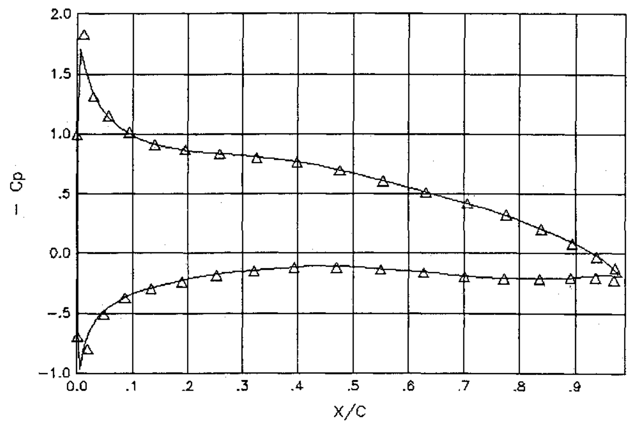

Fig. 5 Pressure coefficient distribution ($\alpha = 4$ deg): — panel method; △ finite element method.

Therefore, the circulation Γ may be calculated uniquely:

$$\Gamma = \frac{(\partial\phi_1/\partial x)_u - (\partial\phi_1/\partial x)_l}{(\partial\phi_0/\partial x)_l - (\partial\phi_0/\partial x)_u} \tag{6}$$

III. Application of a Standard Structural Finite Element Code

The problem described in the previous paragraphs can be solved using a standard structural finite element code. In our work, the MSC/NASTRAN was selected. Using the analogy, the velocity potential ϕ at all grid points was posed equal to the displacement in an arbitrary direction. The remaining degrees of freedom (two displacements and three rotations) were zeroed. Imposing for the stress-strain relationships the same values as described in Refs. 1 and 2, with a null value for the structural density, the structural equations of elasticity become the field equation for the present problem.

The solution requires two different runs. The first evaluates the field ϕ_0, with the proper boundary conditions to simulate a unit circulation over the branch cut. This can be realized by imposing a set of prescribed displacements; on the lower side of the branch cut the displacements are set to 0, on the upper one they are set to 1. The second run is performed imposing the continuity across the branch cut (using the MultiPoint Constraints MSC/NASTRAN cards) in addition to the asymptotic boundary conditions. The latter can be reviewed as forces, along the outward normal of the domain:

Table 1 Characteristics of the runs

Items	Field ϕ_0	Field ϕ_1
Time, s	106.06	146.52
Grid points	902	902
Degrees of freedom	814	880

$$\frac{F}{A} = \frac{\partial\phi_1}{\partial n} = V_\infty \cdot n \tag{7}$$

The model used was a nondimensional domain around a NACA 64_A410 airfoil. The chord of the airfoil was set to 1 (x ranging from -0.5 to 0.5), whereas the dimensions of the domains were 10×10. The mesh contained 902 grid points and 840 QUAD4 elements for the connectivity (Figs. 2 and 3). Table 1 shows the degrees of freedom and the CPU time solution for a standard static analysis (rigid format 24). The analysis was performed on a CDC computer (Model 930/31). A simple postprocessing of the output data will permit the calculation of the additional unknown Γ. The superposition of the two fields was realized to get the velocity potential distribution and, finally, the nondimensional pressure distribution. In Figs. 4 and 5, these results are shown. Also shown are results using a panel method (80 panels); only one other result is shown. It is seen that there is a good correlation.

IV. Conclusions

In this Note, a simple approach to obtain the incompressible, nonviscous, aerodynamic flowfield over a body is presented. The object of the study involves the solution of the well-known Laplace's equation for lifting bodies using a standard structural code, such as MSC/NASTRAN. This Note is the first result obtained in a more general research area concerning the application of the finite element standard code to solve the general (unsteady) fluid-structural coupled problem.

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